Emergent Behavior - a case of Disorder vs. Order

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talk at the Frishman-Fest, Weizmann-Institute,
13. December 1998

System A network of *objects* with directed relations between them which have adjoints in the opposite directions; some of these relations are declared fundamental, or *links.* The objects may be composite, i.e. *systems* themselves. In a

Dynamical System , *links* mediate local interactions which specify a dynamics. **Ex:** lattice gauge theory.

Emergence : Nonlocal phenomena spring from local interactions in a *dynamical system.*

Self organization in a dynamical system: The *emergence* of *composite object*s whose constituents behave in a coherent fashion. This is called

Emergent behavior : Highly structured collective behavior emerges from the local interaction of simple subsystems (⇒ perform a *function*).

Examples of emergent behavior

(from Crutchfield, Physica **D** **75** 11-54 (1994)

schools of fish swimming in a coherent array abruptly turn together with no leader guiding the group

flocks of birds (similar)

emergent global information processing in financial markets, leading to the fixation of prices.

A simple model for the school of fish

Let us ignore the motion in space, and regard the velocity as a $(n = 3)$ -dimensional unit vector - call it *spin*.

The dynamics of the spins has three ingredients:

1. A tendency to "go forward" (retain the spin)
2. A tendency to align with the others
3. If a fish has made an improbably large turn, it is regarded as *"excited"*. An excited fish is perceived more strongly by its unexcited neighbours, and perceives its unexcited neighbours less strongly. Therefore the unexcited fish align preferably with excited fish. This leads to a shock wave.

The improbably large turn may be caused by an external perturbation (e.g. a predator: all fish in his neighbourhood turn away from him) or occur spontaneously (very rarely).

Another model: Fashionable people.

Consider a society of fashionable people, who always want to follow the latest trend. If a trend is established by a sufficiently large group of followers, those in contact with them will convert to the new trend. Conversion to a new trend prevents for a certain time still another switch to a newer trend. The trend will expand in a shock wave fashion, like a forest fire.

Shock waves.

Shock waves occur in excitable media, where a temporary local excitation of the medium is followed by a *dead time* or refractory phase, during which the medium cannot be excited again.

Example: Forest fire. After the vegetation has burned, it takes some time to grow again. During this time no new fire wave may pass.

System theoretic model of shock fronts: Pairs of adjoint links $b = \rightarrow$ and $b^* = \leftarrow$ can exist in 3 states,

1. $(b, b^*) =$ (normal , normal)
2. $(b, b^*) =$ (excited, suppressed)
3. $(b, b^*) =$ (suppressed, excited)

An excited link represents a “strengthened” coupling, and a suppressed link a weakened coupling, e.g. weakened to no coupling at all. The “strengthened” coupling leads to an action of its source S, by which an excitation of all normal links whose target is S is effected. The weakening of a link prevents reaction back.

Shock waves play a central role in the *theory of enzymatic computation*

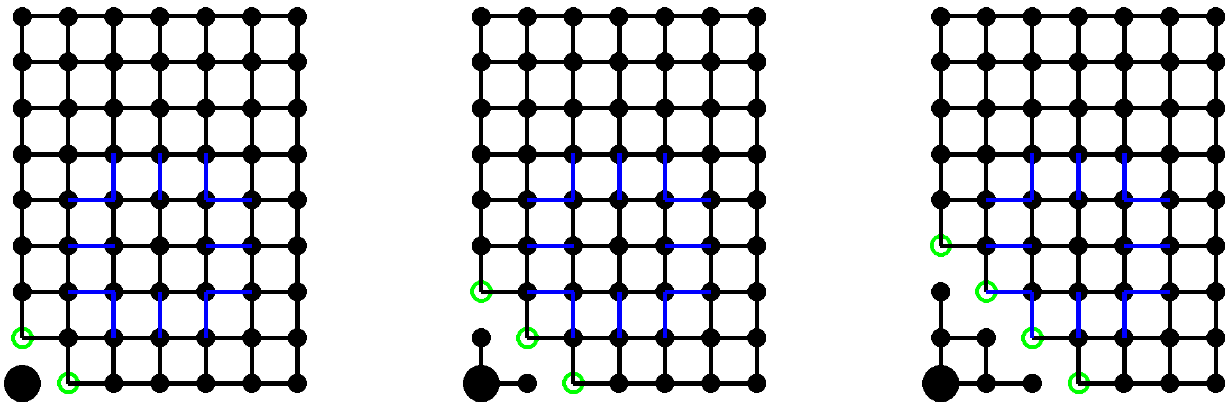
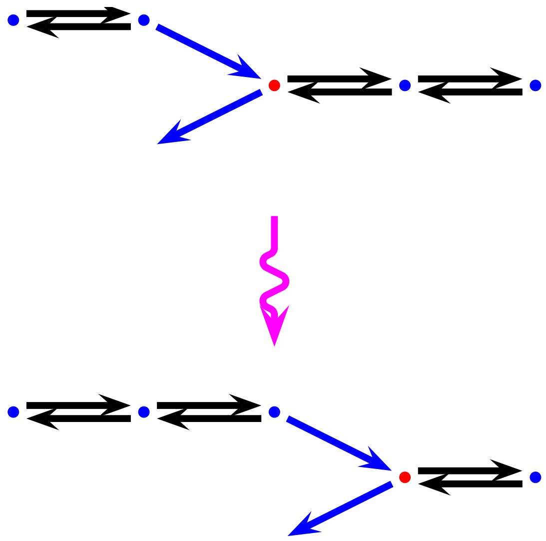


Figure 1: Shock fronts traveling through a matrix of intelligent material - cp. (natural) neural nets. Membranes (blue) can confine the shock to domains inside or outside.



Updating fork dynamics:

This is used for updating via shock waves

Nonequilibrium Thermodynamics as Equilibrium Statistical Mechanics

in one more dimension (real time).

Consider a system in D dimensions with microscopic states ϕ_t at one time $t = 0, 1, ...$

Suppose there is a stochastic dynamics given by transition probabilities

$$0 \leq W(\phi^{t+1}, \phi^t) \quad < \quad \infty \quad \text{such that} \tag{1}$$

$$\int d\phi' W(\phi', \phi^t) \quad = \quad 1. \tag{2}$$

States at one time are given by probability distributions

$\rho_t(\phi^t)$, and

$$\rho_{t+1}(\phi') = \int d\phi^t W(\phi', \phi^t) \rho_t(\phi^t). \tag{3}$$

The stochastic dynamics might come from a deterministic dynamics by coarse graining of dynamical variables.

Regard the microscopic states $\phi = (\phi^t, t \geq 0)$ *sub specie aeternitatis* as microstates of a *statistical mechanical system in equilibrium* in $D+1$ dimensions, with *normalized* probability distribution

$$\rho(\phi) = e^{-\mathcal{H}(\phi)} \tag{4}$$

$$\mathcal{H}(\phi) = -\sum_{t \geq 0} \ln W(\phi^{t+1}, \phi^t) \tag{5}$$

$$\int \prod_{t>0} d\phi^t \rho(\phi) = 1 \ , \tag{6}$$

with periodic boundary conditions (for instance) in space, and initial conditions ϕ^0 as boundary conditions on the hyperplane $t = 0$.

Correlations between variables $\phi^t_{\mathbf{x}}$ and $\phi^{t'}_{\mathbf{x}}$ at different time can be interpreted as correlations in this equilibrium state. Dependence on initial states becomes dependence on boundary conditions. The problem of time corrrelations becomes a problem of correlations in an equilibrium state. In general, the system will be anisotropic.

Locality

Suppose at first that the microstates are given by internal states of objects at $\mathbf{x}$,

$$\phi^t = \{\phi^t_{\mathbf{x}}\}$$

Generalization to variable links (gauge fields) is easy.

Suppose further that the dynamics is local in the sense that the random variable $\phi^{t+1}_{\mathbf{x}}$ is determined by the random variables $\phi^{t+1}_{\mathbf{y}}$, $\mathbf{y} \in N(\mathbf{x})$,

($N(\mathbf{x})$ = neighborhood of $\mathbf{x}$ as determined by the links in the system)

Then the Hamiltonian is local

$$W(\phi^{t+1}, \phi^t) = \prod_{\mathbf{x}} W_{\mathbf{x}}(\phi^{t+1}_{\mathbf{x}}, \phi^t_{N(\mathbf{x})}) \quad (7)$$

$$\mathcal{H} = -\sum_{t,\mathbf{x}} h(\phi^{t+1}_{\mathbf{x}}, \phi^t_{N(\mathbf{x})}) \quad (8)$$

$$h = -\ln W_x \quad (9)$$

Observation: If the *unnormalized* transition probabilities are local,

$$W(\phi^{t+1}, \phi^t) = Z(\phi^t)^{-1} \prod_{\mathbf{x}} T_{\mathbf{x}}(\phi^{t+1}_{\mathbf{x}}, \phi^t_{N(\mathbf{x})})$$

then the normalization does not destroy the locality,(but next-to-nearest neighbour interactions appear.)

$$Z(\phi^t) = \prod_{\mathbf{x}} Z_{\mathbf{x}}(\phi^t_{N(\mathbf{x})}), \tag{10}$$

$$Z_{\mathbf{x}}(\phi^t_{N(\mathbf{x})}) = \int d\phi'_{\mathbf{x}} W_{\mathbf{x}}(\phi'_{\mathbf{x}}, \phi^t_{N(\mathbf{x})}) \tag{11}$$

School of fish as a ferromagnet in 4 dimensions

The Hamiltonian will have the form

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{shock}$$

where $\mathcal{H}_0$ represents the tendency of the fish to align, and H_{shock} embodies the possibility of shock waves. Both terms are local.

Ignore the shocks at first. Postulate unnormalized transition probabilities

$$\begin{aligned} T^0_{\mathbf{x}}(\phi^{t+1}_{\mathbf{x}}, \phi^t_{N(\mathbf{x})}) &= \exp\left(-\phi^{t+1}_{\mathbf{x}} \cdot S^t_{\mathbf{x}}\right) && (12) \\ S^t_{\mathbf{x}} &= \beta_D \phi^t_{\mathbf{x}} + \beta_N \sum_{\mathbf{y}\in N(\mathbf{x})} \phi^t_{\mathbf{y}}, && (13) \\ b_D &\geq \beta_N z && (14) \\ z &= (\text{no. of nearest neighbours}) && (15) \end{aligned}$$

Parametrize the n-dim. unit spins by vectors $\xi_{\mathbf{x}} \perp \Phi$, $\Phi = (1, \mathbf{0})$,

$$\begin{aligned} \phi_{\mathbf{x}} &= (1 - \xi^2_{\mathbf{x}})^{1/2}\Phi + \xi_{\mathbf{x}} && (16) \\ \xi_{\mathbf{x}} &= (2\pi)^{-n/2} \int d\omega d\mathbf{k}\ \tilde{\xi}(\omega, \mathbf{k}) e^{-i\omega t + i\mathbf{k}\mathbf{x}}. && (17) \end{aligned}$$

Evaluate the normalization factors - they produce couplings at one time which are not ferromagnetic. But, for large β

one can expand to order ξ^2, and the Hamiltonian becomes

$$H = \int d\omega d\mathbf{k}\; E(\omega,\mathbf{k})|\tilde{\xi}(\omega,\mathbf{k})|^2 \tag{18}$$

$$E(\omega,\mathbf{k}) = \frac{\hat{\omega}^2}{2}\left[\beta_D + \beta_N z - \beta_N \mathbf{k}^2\right] + \frac{\beta_N^2}{\beta_{tot}}(\mathbf{k}^2)^2 \tag{19}$$

$$\geq 0. \tag{20}$$

$$\beta_{tot} = \beta_D + \beta_N z. \tag{21}$$

For $\beta_D > \beta_N z$ there is no zero mode except $(\omega = 0, \mathbf{k} = 0)$. So this is a ferromagnet!

Spontaneous symmetry breaking

For large β_{tot}, the symmetry under spin-rotations is spontaneously broken (in 3 dimensions), and we have order.

This means that the fish tend to swim in parallel and in the same direction at all times, in the absence of the shock wave term.

Symmetry restoration by the shock wave term

The shock wave term prevents spontaneous symmetry breaking and restores disorder. As a result, in the long run, the fish (in an ∞ ocean) swim in all direction with equal probability.

The correlation time can be very large, if shock waves are triggered by very improbable spontaneous events. We get long correlation time without fine tuning!

Implementation: One attaches extra variables to the links (between neighbouring fish), i.e. a dynamical *gauge field.* Pairs of links may assume the following states

1. $(b, b^*) =$ (normal , normal)
2. $(b, b^*) =$ (excited, suppressed)
3. $(b, b^*) =$ (suppressed, excited)

This changes the coupling strengths,

$$\beta_N^{supp} \approx 0, \tag{22}$$

$$\beta_N^{exc} >> \beta_N^{normal}. \tag{23}$$

Call a fish unexcited if it is not the target of an excited link. The shock wave term arises from a factor in the unnormalized transition probability which affects links as follows:

1. Non-normal links return to normal in the next time step.
2. An improbably large turn of an unexcited fish f excites the links of which it is the source, making the neighbours n at the other end more susceptible to f's direction: $f \mapsto n$

Conclusion: The emergent behaviour is a case of symmetry restoration by a perturbation. Without the perturbation there would be spontaneous symmetry breaking.

A seemingly new problem can be tackled by old and familiar concepts from quantum field theory and statistical mechanics

Entropy production

The reinterpretation also permits to study approach to equilibrium (of the system at one time). The entropy at time t can be defined as

$$S(t) = \int \prod_{\mathbf{x}} d\phi^t_{\mathbf{x}} \rho_t(\phi^t) \ln \rho_t(\phi^t).$$

Such quantities are difficult to compute. But ρ_t are like Schrödinger functionals, and a perturbative computation may be feasible.